DIMENSIONLESS SPECIFIC ROTATIONAL SPEED APPLIED TO TURBOPUMPS

Dr. Podalyro Amaral de Souza[1]
Dra Yvone de Faria Lemos De Lucca[2]

ABSTRACT

This paper describes the study of turbo pumps with single suction, where are applied the similarity laws for hydraulic machines and are defined and analyzed two important dimensionless parameters: specific rotational speed (ω_s) the specific diameter of the impellers (D_s). The objective of this study was the definition of these parameters and the determination of a fitting curve relating the specific diameter of the impellers in function the specific rotational speed. There were collected approximately 200 set of points from catalogues of designers of pumps. For each rotational speed fixed, there were read the values of the head (H*) and discharge (Q*) at a point of maximum efficiency, and then calculated the parameters D_s e ω_s. The specific speed for centrifugal pumps, defined as $n_s = 3{,}65.\frac{n.\sqrt{Q}}{H^{3/4}}$ used in the design of pump plants, is not a dimensionless parameter. One universal characteristic curve for pumps in dimensionless form is the plot of specific rotational speed ω_s as abscissa, against D_s as ordinate. This curve obtained by using least square fitting, can be converted to the usual characteristic curve by selecting the desired values of ω_s and D_s. This article shows the advantages of these dimensionless numbers application.

Key Words: dimensionless analysis, specific rotational speed, specific diameter.

INTRODUCTION

This paper describes the study of turbo pumps with single suction, where are applied the similarity laws for hydraulic machines and are defined the dimensionless parameters as: specific rotational speed (ω_s) and the specific diameter of the impellers (D_s).

PURPOSE

The objective of this study was:

· Develop a detailed analysis of the performance attributes of turbo pumps.

· Develop a detailed dimensional analysis of turbo pumps.

Demonstrate a simplified method to classify turbo pumps using the dimensionless parameter (ω_s).

· Analyse a wide range of turbo pumps produced in Brasil, establish a correlation between the specific diameter of the impellers and the specific rotational speed, both dimensionless, to pre select an adequate turbo pump for a desired application.

SPECIFIC SPEED

One of characteristics to describe the hydraulic type of water turbines which was later applied to centrifugal pumps is called specific speed, which is defined as

$$n_s = 3{,}65.\frac{n.\sqrt{Q}}{H^{3/4}} \quad (1)$$

Where *n* is revolutions per minute, *Q* is the discharge in m3/s and *H* is the head in meter. The physical meaning of the quantity n_s is apparent from the foregoing reasoning: it is the rpm of a standard pump homologous with a given pump and generating, under similar operating conditions, a head is $H_s = 1m$ at a rate of discharge of $Q_s = 0{,}075 m^3/s$. The hydraulic and volumetric efficiencies of the two pumps are, naturally, the same. The physical meaning of specific speed has no practical value and the number is used as a "type"number. The specific speed as a type number is constant for all similar pumps and does not change with the speed for the same pump. Each specific speed are associated with definited proportions of leading impeller dimensions such as b_2/D_2 or D_1/D_2.

In the study of pump performance and classification of all important design constants, specific speed is a criterion of similarity for centrifugal pumps in the manner that Reynolds number is a criterion for pipe flow. When used as a type number, specific speed is calculated for the best efficiency point. For a multistage pump, specific speed is calculated on the basis of the head per stage. When the specific speed of a double suction impeller is compared with that of a single suction impeller the discharge of the first should be divided by 2 or its specific speed should be divided by $\sqrt{2}$. All important pump design and performance characteristics are so closely connected with the specific speed that it is impossible to discuss certain features without reference to it. From equation (1) it follows that, for the same head discharge requirements, higher specific speed pumps will run at a higher speed and will be of smaller physical dimensions. Also, for the same speed and discharge, higher specific speed pumps will operate at a lower head or, for the same speed and head, a higher specific speed pump will deliver a higher discharge.

DIMENSIONLESS CLASSIFICATION

Many pumps are too large to be tested in a laboratory therefore it is essential to be able to test small geometrically similar ones then scale the results up to the required size. Like any other scaling problem in fluid mechanics this is achieved by scaling up dimensionless variables.

Geometric similarity: Geometric similarity is a prerequisite of dynamic similarity. and must be preserved for all the hydraulically important part of the pump, for example entrance and discharge passages, impeller and diffuser (if one is fitted). Machines which are geometrically similar in these respects from a homologous series.

Dynamic similarity: dynamic similarity means a fixed ratio of forces, to achieve this kinematic similarity is also required i.e a ratio of velocities. In practice this means that the inlet and outlet velocity triangles must be geometrically similar.

Analysis: The variables in the analysis are as follows:

H - difference of head across the machine (L);

ω - rotational speed (T-1);

P - power transferred from impeller to fluid (ML2T-3);

g - gravitational acceleration (LT-2);

ρ - density of the fluid (ML-3);

μ - dynamic viscosity of the fluid (ML-1T-1);

Q -discharge through the pump (L3T-1);

D - diameter of the impeller (L);

The functioning of the turbo pump, for a constant rotational speed can be defined by the three law as follows:

H=H(Q),

P=P(Q) and

$\eta = \eta(Q)$

Generically, the functioning of a turbo pump, can be treated using dimensionless groups, considering the fact that the 9 variables related to each other, can determine the pump operation:

$$\phi_0(g, H, Q, P, D, \rho, \omega, \upsilon, \eta) = 0 \quad (2)$$

The following dimensionless group can be obtained when the variables

$$\phi_1\left(\frac{g}{\omega^2 D}, \frac{H}{D}, \frac{P}{\omega^3 D^5}, \frac{Q}{\omega D^3}, \frac{\upsilon}{\omega D^2}, \eta\right) = 0 \quad (3)$$

The above functional relationship can be simplified as following:

$$\phi_2\left(\frac{gH}{\omega^2 D^2}, \frac{P}{\omega^3 D^5}, \eta, \frac{Q}{\omega D^3}\right) = 0 \quad (4)$$

In the group of dimensionless variables in the equation (4) is not present the viscosity υ which was undervalued because of the high turbulent flow within the impeller and the volute. The dimensionless parameter including "g" and "H" were multiplied, to create a new dimensionless number.

A centrifugal pump can be represented by universal characteristic curves:

$$C_H = C_H(C_Q) \quad (5)$$

$$C_P = C_P(C_Q) \quad (6)$$

$$\eta = \eta(C_Q) \quad (7)$$

where

$$C_H = \frac{g.H}{\omega^2 D^2} \quad \text{(head coefficient)} \quad (8)$$

$$C_P = \frac{P}{\rho.\omega^3 D^5} \quad \text{(power coefficient)} \quad (9)$$

$$C_Q = \frac{Q}{(\omega D^3)} \quad \text{(discharge coefficient)} \quad (10)$$

PROCEEDINGS

There were collected pumps technical data from several different pump manufactures (single suction pumps) as follows: KSB, Haupt do Brasil, Flygt do Brasil , Flowserve, Imbil, and Weir do Brasil. From each pump type, it was read the pair of points (H, Q) correspondent to the maximum efficiency, along with their correspondent rotational speed and diameter of the impeller. The data came from radial flow pumps, mixed flow pumps and axial flow pumps. The range of the head was $2{,}05 \le H \le 76{,}0$ m; the range of the discharge was $0{,}00161 \le Q^* \le 1{,}880$ m3/s. For pumps with double suction impeller the specific rotational speed were calculated considering the discharge divided by 2.

ANALITICAL TREATMENT

To obtain a dimensionless parameter which includes ω, H, Q and D we can combine the discharge coefficient and the head coefficient; this is possible because all the π groups are dimensionless therefore we can divide them as shown below:

$$D_s = \left\{\frac{(C_H)}{(C_Q)^2}\right\}^{1/4} = \left\{\frac{\frac{g.H}{\omega^2.D^2}}{\left(\frac{Q}{\omega D^3}\right)^2}\right\}^{1/4} \quad (8)$$

The simplification form of equation (8) is:

$$D_s = D.\frac{(g.H)^{1/4}}{\sqrt{Q}} \quad (9)$$

The specific rotational speed ω_s was defined as:

$$\omega_s = \left\{\frac{(C_Q)^2}{(C_H)^3}\right\}^{1/4} = \left\{\frac{\left(\frac{Q^*}{\omega.D^3}\right)^2}{\left(\frac{g.H^*}{\omega^2.D^2}\right)^3}\right\}^{1/4}$$

$$\omega_s = \frac{\omega.\sqrt{Q^*}}{(g.H^*)^{3/4}} \quad (10)$$

where ω is the angular velocity which is related to "n (rpm)" as:

$$w = \frac{2.\pi.n}{60} \quad \text{(rad/s)} \quad (11)$$

CLASSIFICATION

Centrifugal and other turbo pumps may be classified according to the specific rotational speed (ω_s) as follows:

- Turbo pump with radial flow: $\omega_s \le 1$
- Turbo pump with mixed flow: $1 < \omega_s < 4$
- Turbo pump with axial flow: $\omega_s \ge 4$

The specific rotational speed (ω_s) is always calculated with the values of ω, Q and H at a condition of maximum efficiency.

CORRELATION ANALYSES

To establish a functional relationship between D_s and ω_s, or $D_s = D_s(\omega_s)$, it was used the method of least squares, in an Excel Plan. To better adjust of the equation it was adopted a functional relationship $D_s = D_s(\omega_s.e^{\omega_s})$ since that $\omega_s \le 1$ and the resulted determination index ended close to 1. It was obtained the function in the form of an exponential function;

$$D_s = 5{,}426608.(\omega_s.e^{\omega_s})^{-0{,}592987} \quad \omega_s \le 1 \quad (12)$$

with the determination index $R^2 = 0{,}974153$

For the range $1 < \omega_s \le 5{,}1$ the equation resulted by least squares method was of the form an exponential function, as follow below: $D_s = 2{,}906788.(\omega_s)^{-0{,}455205}$ for

$$1 < \omega_s \le 5{,}1 \quad (13)$$

with the determination index $R^2 = 0{,}810244$

RESULTS

The results are shown in the Figures 1and 2 below:

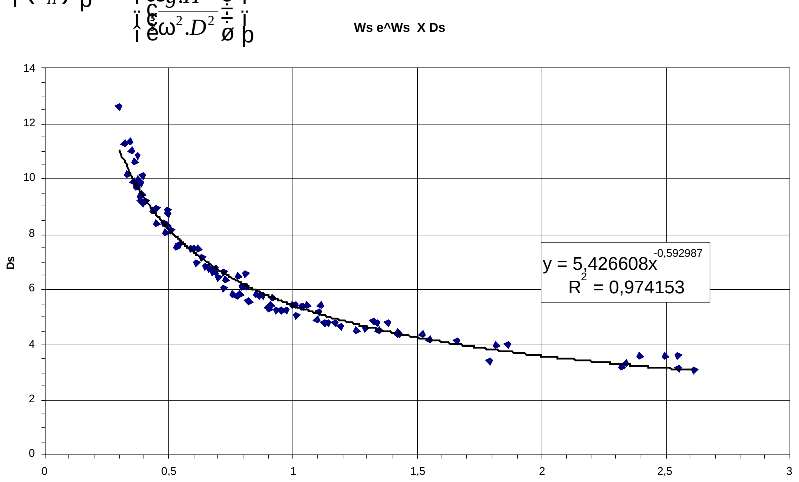

Figure 1- Functional relationship: $D_s = D_s(\omega_s.e^{\omega_s})$ for $\omega_s \le 1$

1 - EPUSP- podalyro@usp.br
2 - FAAP - yvone@cth.usp.br / yvone.lucca@gmail.com

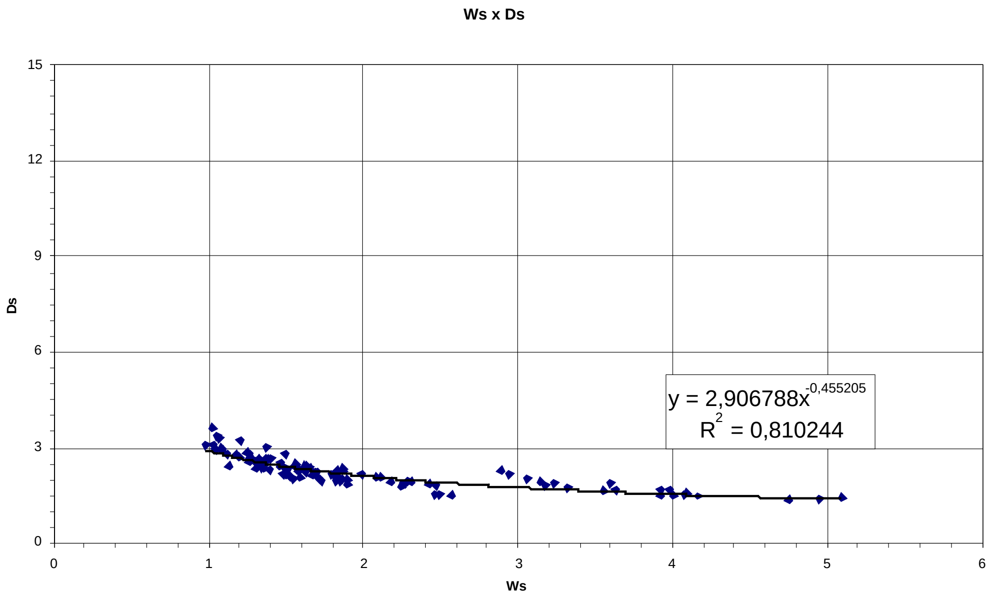

Figure 2- Function relationship $D_s = D_s(\omega_s)$ for $1 < \omega_s \leq 5{,}1$

CONCLUSIONS AND RECOMMENDATIONS

• The proposal to classify turbo pumps using a specific rotational speed " ω_s" , showed to be accurate and easiest to use than the specific speed " n_s".

• The use of specific rotational speed " ω_s" is then preferable since it is a dimensionless parameter.

• It is relevant to say that specific rotational speed " ω_s" in the transition of radial to mixed flows assumes value equal to 1, which is an indication of a well defined dimensionless parameter, as Mach´s number and Froude´s number.

• The dimensionless parameter ω_s was then classify according to the range below:

Centrifugal pumps have low specific speeds, $\omega_s \leq 1$

Mixed flow pumps have medium specific speeds, $1 < \omega_s < 4$

Axial flow pumps have high specific speeds, $\omega_s \geq 4$

• An engineer involved with pump station design can use with advantage the results of this paper: the design values of discharge Q, head H and rotation of the shaft n (rpm) can be take as associated to maximum efficient point; the n (rpm) is then transformed into ω (rad/s), using equation (11); the specific rotational speed (ω_s) is then obtained from equation (10) and the flow through the impeller can be defined; the value of ω_s in equation (12) or (13) gives the correspondent value of D_s, from which can be calculated the appropriated impeller diameter for the design.

· In the future, this study can be extended for pumps with a double suction and multistage pump.

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